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STUDY ABOUT DAMPING RATIO OF THE HUMAN BODY - SEAT SYSTEM

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Abstract. This paper investigates the biodynamic response of human body subjected to vertical vibrations into an auto vehicle, in two different situations: the driver sitting on a rigid seat and respectively the driver sitting on a vehicle seat with seat cushion and additional seat suspension. The eigenvalues and natural frequencies of both systems are calculated. The biodynamic model is found to belong to a damped system, by calculating the damping ratio matrix of the model. The stability of the model is identified by solving the eigenvalues problem of the biodynamic system.

1. INTRODUCTION

Since the human body behaves like a vibrating physical system, an appropriate mathematical model can be developed using its mass, elastic and damping properties. Most current models are MDOF models, which represent the human body by combining masses, springs, and dampers, corresponding to the different body parts. The natural frequencies of a vibrating system are the most important parameters needed to study the steady-state and dynamic behavior of the system. The concept of the eigenvalues of a matrix is closely related to the concept of natural frequency of vibration in a vibrating system. An undamped n-DOF system has *n* number of natural frequencies. A damped n-DOF system also will have *n* number of natural frequencies which are complex and some of them, however, may have over damped modes. The classification of critical damping, over damping, and under damping, for the system can be carried out by examining the damping ratios or damping ratio matrix of the system.

2. BIODYNAMIC MODELING

There are two broad categories of biodynamic models: continuous-parameter models and lumped-parameter models. Considering the human body as a *mechanical system*, at low frequencies (less than 100 Hz) and low vibration levels, it may be roughly approximated by linear lumped parameter systems. In lumped-parameter models, various characteristics in the system are lumped into representative elements. In an analytical model, these individual characteristics can be approximated by a separate mass element, a spring element, and a damper element, which are interconnected in parallel or series configuration. In terms of the component energy, the basic system elements can be divided into two groups: *energy-storage elements* (i.e. masses and springs) and *energy* dissipation elements (i.e. dampers). When human body is subjected to vibration, different parts of the body move relative each to other. Thus, the body behaves like a vibrating physical system with distributed energy-storage elements (masses, springs) and energy-dissipation elements (dampers). In order to study the biodynamic behavior of different segments in the body, it is possible to model the body system by a MDOF lumped-parameter biodynamic system. The degrees of freedom and the structure of the model are based on the general trends observed from the measured biodynamic data, and available models, instead of knowledge of anatomy and anthropometrics. The 4-DOFs linear biodynamic model of human body proposed by Zong & Lam (2002) is selected to study the vertical vibrations transmitted to the human body inside an auto vehicle (Fig.1). The model comprises four masses, coupled by linear elastic and damping elements. The four masses represent the following four body segments for a seated individual: pelvis, upper torso, viscera and head. For a seated driver, the weight distribution on the seat may vary with the sitting posture adopted, such as erect or slouched positions, with or without backrest support. The mass due to lower legs and feet is not included in this representation, assuming that their contribution to the biodynamic response of the seated body is negligible. Similarly, the hand and arm mass supported on the steering wheel is considered negligible. The stiffness and damping properties of the various body segments are represented by the constants $\boldsymbol{k_i}$ and $\mathbf{c_i}$, respectively. The parameters of this model are given as follows [6]:

M_2 = 29 kg	C_2 = 108.42 Ns/m	K_2 =16.21 ·104 N/m
M_3 = 21.8 kg	C_3 = 199.72 Ns/m	K_3 = 3.78 ·104 N/m
M_4 = 6.8 kg	C_4 = 138.74 Ns/m	K_4 = 0.28 ·104 N/m
M_5 = 5.5 kg	C_5 = 210.95 Ns/m	K_5 = 20.22 ·104 N/m

$\Sigma \mathbf{M_i}$=63.1 kg

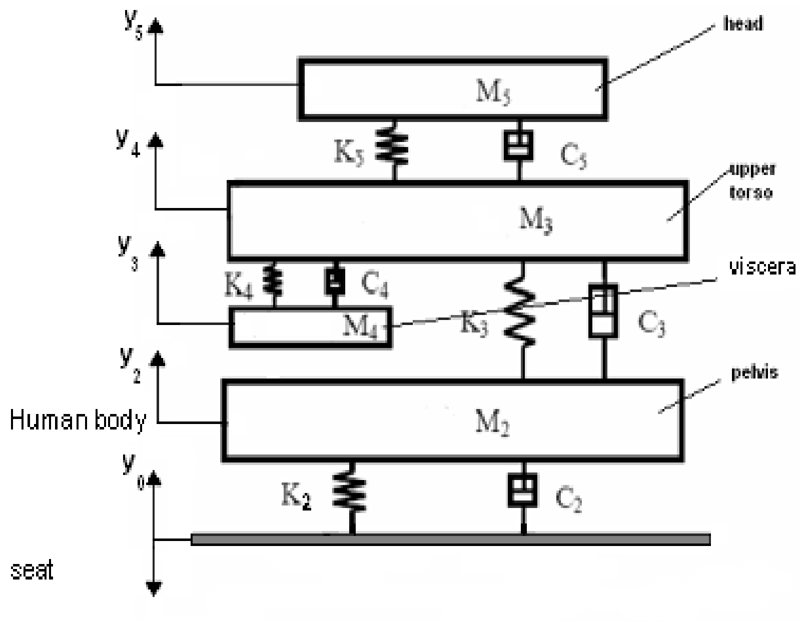

Fig. 1 Mechanical model of the human body [6] sitting upright on the Rigid Seat acted upon by vertical harmonic displacement y_s

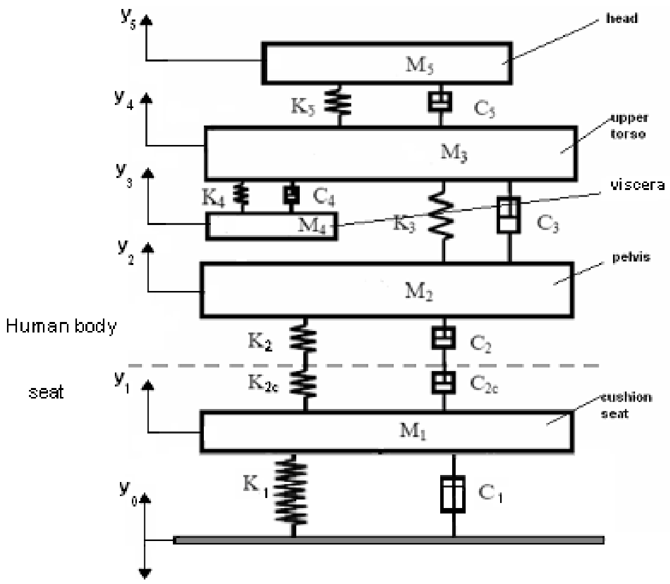

Fig. 2 Mechanical model of the human body sitting upright on the seat cushion acted upon by vertical harmonic displacement y_s [5]

3. EIGENVALUES AND NATURAL FREQUENCIES

The natural frequencies are the main factors influencing human response to WBV. The concept of the eigenvalue of a matrix is closely related to be concept of natural frequency of the vibration in dynamic structures, just *as* the roots of the characteristic equation and natural frequency of a SDOF system are related. For a MDOF system therefore, the natural frequencies can be determined by solving the eigenvalue problem of the system.

In this study, there are a comparison between the eigenvalues of the system made by the human sitting upright on the rigid seat (RS) and the eigenvalues from the same mechanical model of the human body sitting upright on the seat cushion (SC), represented by mass M_1, spring K_{2c} and damping $C_{2c.}$ The soft seat cushion was implemented as a linear spring K_{2c} and damper C_{2c} system. The mass of the moving part of the seat, M_1, was estimated at about 13,5 kg. The seat is fixed to the floor through the seat suspension which is formed by the spring and dashpot and is represented by the spring $K_1=2.26\cdot10^4$ N/m and damping $C_1=750$ Ns/m (Fig. 2).

3.1. The motion differential equations

The system (Fig. 1 and 2) contains four or five independent parameters and the general differential equations system can be written in matrix form (1):

$$[M]\{\ddot{Y}\}+[C]\{\dot{Y}\}+[K]\{Y\}=[D]\{\dot{Y}\}+[S]\{Y\} \tag{1}$$

where **[M]** - matrix of inertia, **[C]** - matrix of damping coefficient, **[K]** - stiffness matrix, $\{Y\}$ - the elements displacements; $\{\dot{Y}\}$- velocities, and $\{\ddot{Y}\}$- accelerations of the same elements and **[D]** – stiffness matrix and respectively **[S]** matrix of damping coefficient given by excitation signal.

Table 1 presents the eigenvalues obtained from the homogeneous form of equations.

In both cases all the complex natural frequencies (eigenvalues) have imaginary parts indicating that the system can perform free oscillations at frequencies 2.311 Hz, 3.12 Hz, 8.537 Hz and at 34.033 Hz respectively 2.063 Hz, 3.052 Hz, 8.321 Hz, 9.379 Hz and at 34.033 Hz.

The real parts of all these complex natural frequencies have negative values and belong to the left side of the imaginary axis (vertical axis). This means that the system under study has a dynamic stability.

Table 1 Eigenvalues [4]

For Rigid Seat (RS)	For seat with cushion and suspension (SC)
λ1,2 = -4.093±34.033i Hz	λ1,2 = -4.094±34.033i Hz
λ3,4 = -1.464± 8.537i Hz	λ3,4 = -3.504± 9.379i Hz
λ5,6 = -1.792± 3.12i Hz	λ5,6 = -2.469± 8.321i Hz
λ7,8 = -0.177± 2.311i Hz	λ7,8 = -1.844± 3.052i Hz
	λ9,10= -0.296± 2.063i Hz

4. DETERMINATION OF DAMPING RATIOS

The definitions of critical damping, over damping, and under damping in SDOF systems can be extended to MDOF systems. In order to determine the damping ratios (or damping matrix) of MDOF systems, the expression for the damping ratio for SDOF model is first derived.

4.1. Damping ratio matrix for MDOF system

For a MDOF system, the equations of motion may be written in a matrix form as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=0 \tag{2}$$

Since the matrix M is positive definite, it has a positive definite square root. That is, there exists a unique positive definite matrix $\boldsymbol{M^{1/2}}$ such that $\boldsymbol{M^{1/2}\cdot M^{1/2}=M}$. The eigenvalues of $\boldsymbol{M^{1/2}}$are $\boldsymbol{\beta_i}^{1/2}$, where $\boldsymbol{\beta_i}$ are the eigenvalues of $\boldsymbol{M}$.

Thus, equation (2) is rewritten as:

$$\{\ddot{x}\}+M^{-1/2}CM^{-1/2}\{\dot{x}\}+M^{-1/2}KM^{-1/2}\{x\}=0 \tag{3}$$

The difference between $M^{-1/2}CM^{-1/2}$, $M^{-1/2}KM^{-1/2}$ and $\mathbf{M^{-1}C}$, $\mathbf{M^{-1}K}$ is that the matrices $M^{-1/2}CM^{-1/2}$, $M^{-1/2}KM^{-1/2}$ are symmetric and positive definite, whereas $\mathbf{M^{-1}C}$, $\mathbf{M^{-1}K}$ are not necessarily symmetric.

Following the treatment of the SDOF system, introduce

$$\begin{aligned} C_s &= M^{-1/2}CM^{-1/2} \\ K_s &= M^{-1/2}KM^{-1/2} \end{aligned} \tag{4}$$

In a form imitating the SDOF case, a critical damping matrix is defined to be:

$$C_{cr} = 2K_s^{1/2} \tag{5}$$

The damping ratio matrix for a MDOF model is defined by:

$$Z = C_{cr}^{-1/2} C_s C_{cr}^{-1/2} \tag{6}$$

Furthermore, define the matrix Z_{diag}, to be the diagonal matrix of eigenvalues of the matrix **Z**, *i.e.(Inman* (1990) [2].

$$Z_{diag} = diag[\lambda_i, (Z)] = diag[\zeta_i^*] \tag{7}$$

where the ζ_i^* are modal damping ratios in that if $0 < \zeta_i^* < 1$, the system is under damped, if $\zeta_i^* > 1$, the system is over damped, if $\zeta_i^* = 1$, the system is critically damped, and if ζ_i^* are indefinite, the system is said to exhibit mixed damping.

For equation (3), it may be seen that if $CM^{-1}K = KM^{-1}C$ (normal model case), then $Z = Z_{diag}$,

Then the following classifications [2] can be derived:

- If $\boldsymbol{C_s = C_{cr}}$ (or **[I-Z] =0**, where ***I*** is the identical matrix), then the system is said to be a critically damped system, each mode of vibration is critically damped, and each eigenvalue of the system is a repeated negative real number. The response of such systems will not oscillate, and ail the eigenvectors are real.
- If the matrix $\boldsymbol{C_s - C_{cr}}$ is positive definite *(or **[I-Z]<0***), then the system is said to be an over damped system, each "mode" of the structure is over damped, and each eigenvalue is a negative real number. The response of such systems will not oscillates, and all the eigenvectors are real.
- If the matrix $\boldsymbol{C_s - C_{cr}}$ is negative definite (or ***[I-Z] > 0***), then the system is said to be an under damped system, each mode of vibration is under damped, and each eigenvalue is a complex conjugate pair with negative real part. The response of such systems oscillates with decaying amplitude and the eigenvectors are, in general, complex (unless $CM^{-1}K = KM^{-1}C$ *).*
- A fourth possibility exists for the matrix case. That is, the matrix $\boldsymbol{C_s - C_{cr}}$ (or **[I-Z]**) could be indefinite. In this case the system is said to exhibit mixed damping, and at least one mode oscillates and at least one mode does not oscillate.

4.2. Calculation of damping ratio matrix

Substituting the parameters of the system under study into equations (4) to (5), the critical damping matrix for (RS) is obtained as:

$$C_{cr} = \begin{bmatrix} 308{,}14 & -199{,}72 & 0 & 0 \\ -199{,}72 & 549{,}41 & -138{,}74 & -210{,}95 \\ 0 & -138{,}74 & 138{,}74 & 0 \\ 0 & -210{,}95 & 0 & 210{,}95 \end{bmatrix} \tag{8}$$

The damping ratio matrix is determined as follows, by using equation (6).

$$Z = \begin{bmatrix} 0{,}0135 & 0{,}3730 & 0{,}1294 & 0{,}2645 \\ -0{,}0323 & 0{,}5410 & 0{,}1997 & 0{,}4080 \\ -0{,}0323 & 0{,}6880 & 0{,}3468 & 0{,}4080 \\ -0{,}0323 & 0{,}7228 & 0{,}1997 & 0{,}5899 \end{bmatrix} \tag{9}$$

The matrix ***[I - Z]*** (where ***I*** is the identity matrix) is obtained as:

$$[I - Z] = \begin{bmatrix} 0{,}9864 & 0{,}3730 & -0{,}1294 & -0{,}2645 \\ 0{,}0323 & 1{,}541 & -0{,}1997 & -0{,}4080 \\ 0{,}0323 & 0{,}6880 & 0{,}6531 & -0{,}4080 \\ 0{,}0323 & 0{,}7228 & -0{,}1997 & 0{,}4100 \end{bmatrix} \tag{10}$$

The matrix ***[I - Z]*** is indefinite. Therefore, the system is said to exhibit mixed damping, based on above classifications.

Furthermore, considering

$$Z_{diag} = diag[\lambda_i, (Z)] = diag[\zeta_i^*]$$

it obtained

$$Z_{diag} = \begin{bmatrix} 0{,}0762 & 0 & 0 & 0 \\ 0 & 0{,}4981 & 0 & 0 \\ 0 & 0 & 0{,}1689 & 0 \\ 0 & 0 & 0 & 0{,}1194 \end{bmatrix} \tag{11}$$

In the matrix of **Z** $\mathbf{Z_{diag}}$ all the damping ratio are $0 < \zeta_i^* < 1$, and hence they are under damped. As a result, the model of human body considered here is a damping system. The model has all the damped natural frequencies.

For the other system (SC) the above matrices are:

$$C_{cr} = \begin{bmatrix} 814{,}46 & -64{,}46 & 0 & 0 & 0 \\ -64{,}46 & 264{,}18 & -199{,}72 & 0 & 0 \\ 0 & -199{,}72 & 549{,}41 & -138{,}74 & -210{,}95 \\ 0 & 0 & -138{,}7 & 138{,}7 & 0 \\ 0 & 0 & -210{,}95 & 0 & 210{,}95 \end{bmatrix} \tag{12}$$

$$Z = \begin{bmatrix} 0{,}0774 & -0{,}0064 & -0{,}0539 & 0{,}0187 & 0{,}0382 \\ 0{,}0429 & -0{,}0072 & -0{,}6813 & 0{,}2365 & 0{,}4831 \\ 0{,}0429 & -0{,}0531 & -0{,}8493 & 0{,}3067 & 0{,}6267 \\ 0{,}0429 & -0{,}0531 & -0{,}9963 & 0{,}4538 & 0{,}6267 \\ 0{,}0429 & -0{,}0531 & -1{,}0311 & 0{,}3067 & 0{,}8085 \end{bmatrix} \tag{13}$$

$$[I-Z]=\begin{bmatrix} 0{,}9225 & 0{,}0064 & -0{,}0539 & 0{,}0187 & -0{,}0382 \\ -0{,}0429 & 1{,}0072 & 0{,}6813 & 0{,}2365 & -0{,}4831 \\ -0{,}0429 & -0{,}0531 & 1{,}8493 & 0{,}3067 & -0{,}6267 \\ -0{,}0429 & -0{,}0531 & 0{,}9963 & 0{,}5461 & -0{,}6267 \\ -0{,}0429 & -0{,}0531 & 1{,}0311 & -0{,}3067 & 0{,}1914 \end{bmatrix} \tag{14}$$

$$Z_{diag}=\begin{bmatrix} 0{,}350 & 0 & 0 & 0 & 0 \\ 0 & 0{,}142 & 0 & 0 & 0 \\ 0 & 0 & 0{,}517 & 0 & 0 \\ 0 & 0 & 0 & 0{,}284 & 0 \\ 0 & 0 & 0 & 0 & 0{,}119 \end{bmatrix} \tag{15}$$

For the HB/S system with seat cushion and seat suspension, in the matrix of $\mathbf{Z_{diag}}$ all the damping ratio are $0<\zeta_i^*<1$, and hence they are under damped, too. As a result, the model of HB/S considered here is a damping system.

The system manner display can be shown using the following approximate method.

The undamped modal frequencies are approximately expressed as:

$$\omega_i{}^* = \sqrt{\frac{k_i}{m_i}} \tag{16}$$

The modal damping ratios are approximately calculated by:

$$\varsigma_i{}^* = \frac{c_i}{c_{cri}} = \frac{c_i}{2\sqrt{k_i m_i}} \tag{17}$$

Using the relation (17) for HB/S (SC) system it can be obtained the following modal damping ratios: $\zeta_1=0{,}679$; $\zeta_2=0{,}034$; $\zeta_3=0{,}11$; $\zeta_4=0{,}499$; $\zeta_5=0{,}1$. All the damping ratios are $0<\zeta_i^*<1$.

Consequently, the model of human body / seat system considered here is a ***damped system***.

In fact, from an examination of the damping matrix or damping ratios of biodynamic models, it can be seen that they are under damping systems.

5. CONCLUSIONS

The eigenvalues and natural frequencies, damping ratios and damping ratio matrix, are computed.

The biodynamic model is found to belong to a under damping system, in which all modes oscillate, by calculating the damping ratio matrix of the model.

Solving the eigenvalues problem of the biodynamic system identifies the stability of the model. In fact, it can be easily shown that a given linear system is stable if and only if it has no eigenvalues with positive real part. Moreover, the system will be asymptotically stable if and only if all of its eigenvalues have negative real parts (no zero parts allowed).

The eigenvalues approach to stability has the attraction of being both necessary and sufficient.

Consequently, the 4-DOF biodynamic model is asymptotically stable since the real parts of all of eight eigenvalues are negative, respectively, for 5 DOF model all of tenth eigenvalues are negative.

Adding the seat cushion with the mechanical characteristics (mass, stiffness, damper) and the additional seat suspension the given mechanical system (the human body) modifies its eigenvalues closely the natural frequencies. This means the seat with an additional mechanical system changes the human body natural frequencies and can protect the human body inside an auto vehicle.

Research will continue with a comparative study between the biodynamic responses of HB using some different seat suspensions.

REFERENCES

[1] Griffin M.J. *Handbook of Human Vibration*, Academic Press Ltd, London, 1990

[2] Inman, D.J. and Andry, A.N., *Some results on the nature of eigenvalues of discrete damped linear system*, ASME Journal of Applied Mechanics, 47, 1990, no. 4: 927-930.

[3] Lee, R.A., Pradko, F. *Analytical analysis of human response to vibration,* Technical Paper 680091, Society of Automotive Engineers, Warrendale, PA.1968

[4] Rodean, S., Arghir, M. *Eigenvalues of the human body under the vertical vibrations action into a vehicle*,, Acta Technica Napocensis, 2005, 48 vol.**III**, pag. 133-140, ISSN 1221-5872

[5]. Rodean, S., Arghir, M., *Studies regarding modeling of human body / seat assembly simulation under the vertical vibrations into an autovehicle*, COMEFIM'8, The 8 th International Conference of Mechatronics and Precision Engineering, june 8-10, 2006, Cluj-Napoca, Romania, Acta Technica Napocensis, 2006 , 49 vol.**II**, pag. 403-408, ISSN 1221-5872

[6] Zong, Z., Lam, K.Y.: *Biodynamic response of shipboard sitting subject to ship shock motion*, Journal of Biomechanics 35, Editura Elsevier, 2002.